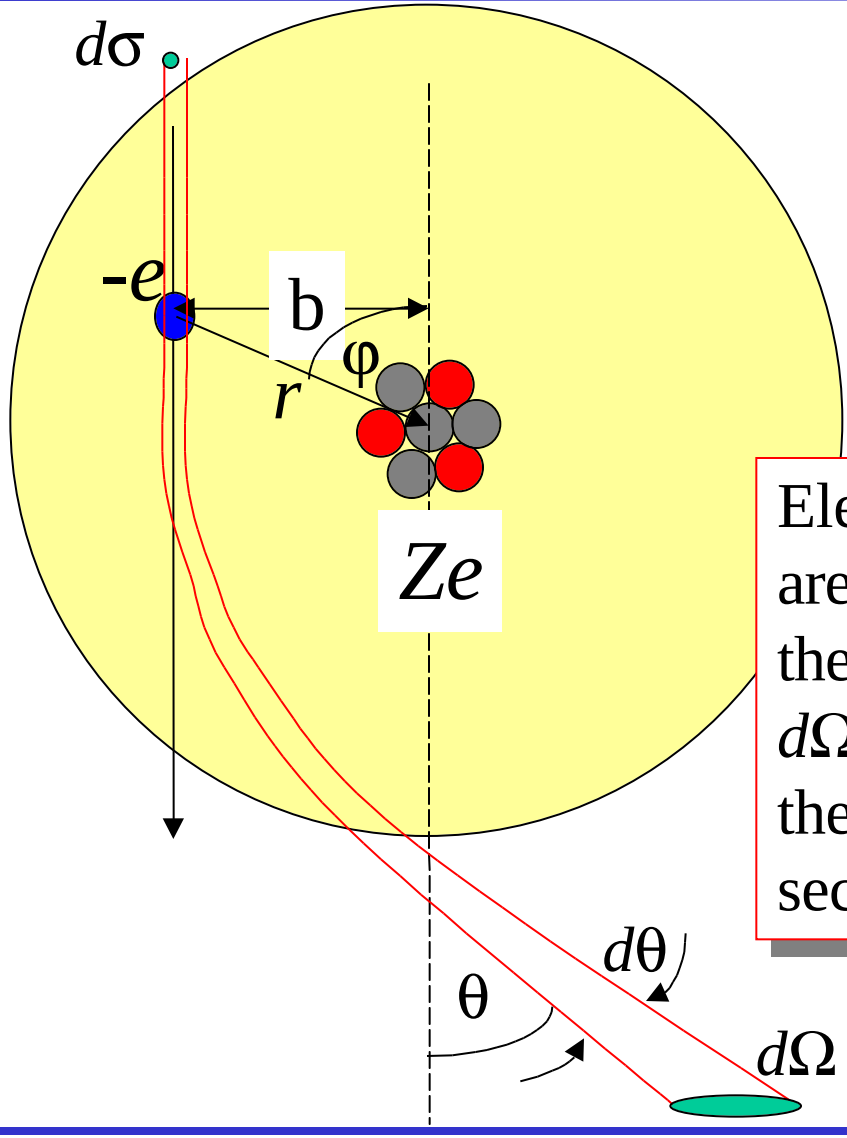
Electron-Solid Interactions

- Where do they go?
  - Elastic scattering
    - Single scattering
    - Plural scattering
    - Continuous slowing down approximation
    - Multiple scattering/diffusion
- What do they do?
  - Inelastic scattering
    - Energy loss mechanisms

Ludwig Reimer, "*Scanning Electron Microscopy*", Springer-Verlag (1985)
Ludwig Reimer, "*Transmission Electron Microscopy*",4th, Springer-Verlag (1997)

Elastic Scattering – Rutherford Cross-section



$$F = -\frac{e^2 Z}{4\pi\varepsilon_0 r^2}\mathbf{u}_r$$

Electrons that pass through the area $d\sigma$ are scattered through the angle θ into a solid angle $d\Omega$. $d\sigma / d\Omega$ is referred to as the differential scattering cross section.

Rutherford Differential Scattering Cross-Section

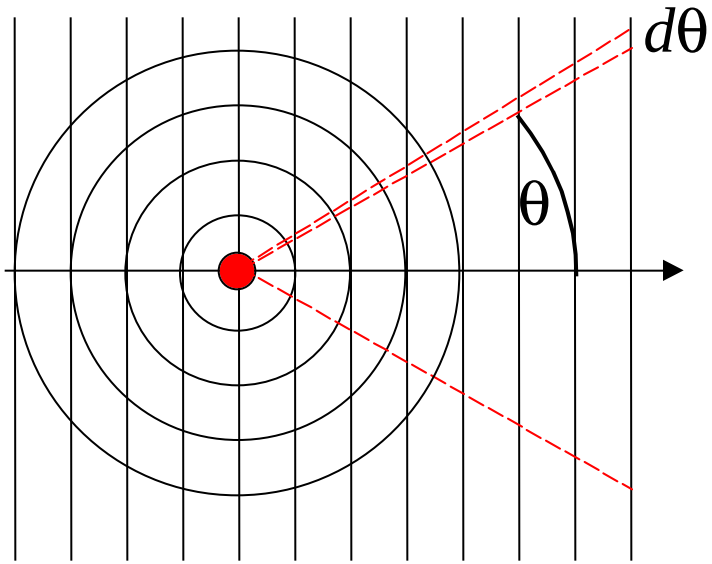
Consider angular momentum, resolve motion into horizontal and vertical components. At the start of the trajectory v_h is zero, at the end $v_h = v\sin\theta$:

$$\frac{d\sigma}{d\Omega} = \frac{e^4 Z^2}{4\left(4\pi\varepsilon_0\right)^2 m^2 v^4} \frac{1}{\sin^4(\theta/2)}$$

- Scattering proportional to Z^2
- Scattering forward peaked
  - Singularity at $\theta = 0$

Elastic Scattering – Other Cross-Sections

- Need to account for screening of nuclear charge by atomic electrons
  - Quantum mechanical treatment considers superposition of plane wave and spherical scattered wave



$$\textit{Plane wave}: \quad \psi = \psi_0 e^{2\pi i k_0 z}$$

$$\textit{Scattered wave}: \quad \psi = \psi_0 f(\theta) \frac{e^{2\pi i k_0 z}}{r}$$

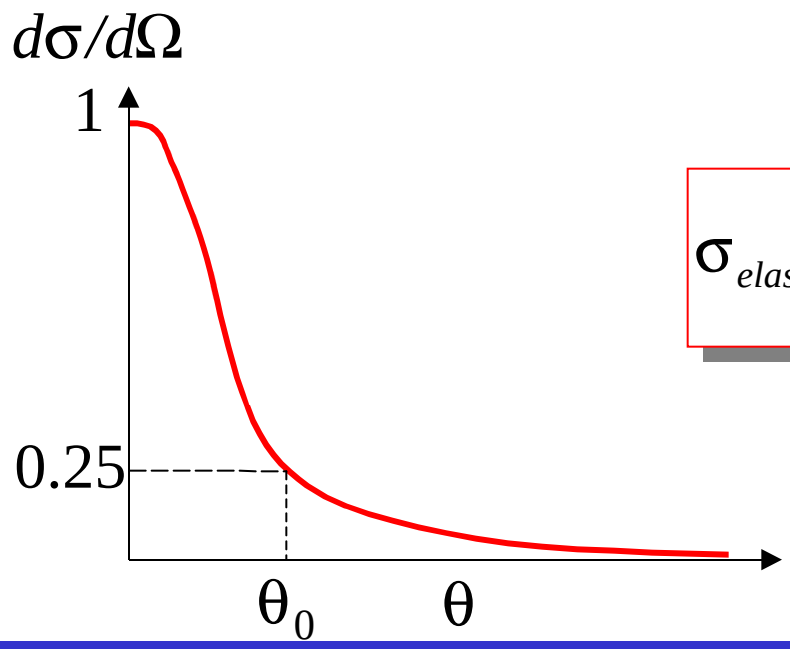
$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

- $f(\theta)$ is the angle dependent scattering amplitude and represents Fraunhofer (far-field) diffraction by the atomic potential
- Numerous cross-sections derived according to form of potential

Screened-Rutherford Cross-Section

$$\frac{d\sigma}{d\Omega} = \frac{4Z^2(1+E/E_0)}{a_H^2}\frac{1}{\left[1+(\theta/\theta_0)^2\right]^2} \quad , \quad \theta_0 = \frac{\lambda Z^{1/3}}{2\pi a_H}$$

θ_0 = characteristic scattering angle, typically 10's mrad at 100 kV



Total Elastic Cross-Section

$$\sigma_{elastic} = \frac{Z^{4/3}\lambda^2(1+E/E_0)^2}{\pi} = \frac{h^2 Z^{4/3}}{\pi E_0^2 (v/c)^2}$$

Mean-Free-Path

$N = atoms / unit\ volume$

$n = atoms / unit\ area$

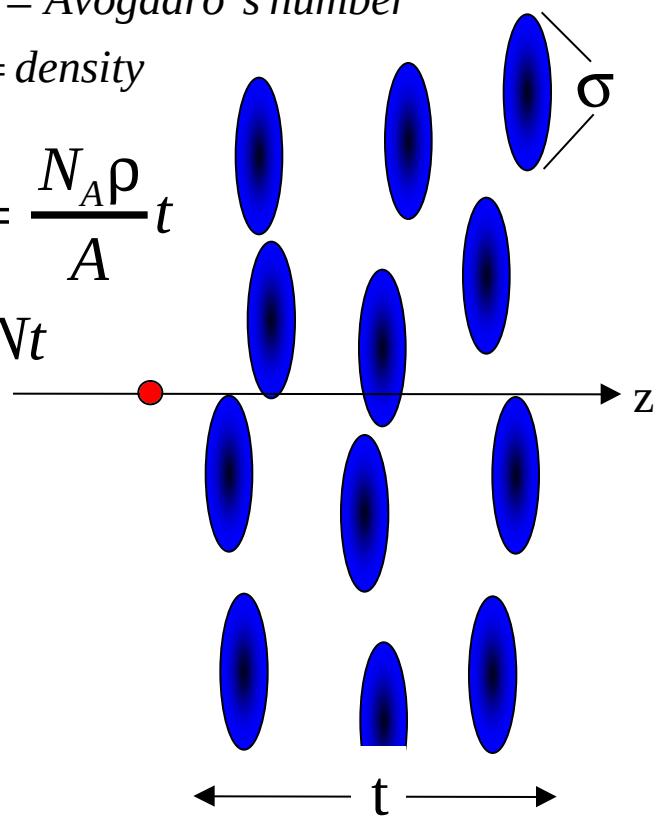
$n\sigma = scattering\ area\ fraction$

$N_A = Avogadro's\ number$

$\rho = density$

$$n = \frac{N_A \rho}{A} t$$

$$= Nt$$



$$dI = -IN\sigma dz$$

$$I = I_0 e^{-N\sigma t} = I_0 e^{-t/\Lambda}$$

$$\Lambda = \frac{1}{N\sigma} = Mean\ Free\ Path$$

$$P(n) = \frac{(t/\Lambda)^n}{n!} e^{-t/\Lambda}$$

$$P(0) = e^{-t/\Lambda} = fraction\ unscattered$$

Element	Z	$\Lambda_{50\,kV}$ (nm)	R (μm)
C	6	83	22.6
Al	13	49	16.7
Cu	29	10.7	5.1
Ag	47	7.7	4.3
Au	79	4.6	2.3

Angular Distribution & Beam Broadening

$$I_1(\theta)d\Omega = I_0 n \frac{d\sigma}{d\Omega} d\Omega = I_0 N\sigma t\left(\frac{1}{\sigma}\frac{d\sigma}{d\Omega}\right)d\Omega = I_0 \frac{t}{\Lambda} S_1(\theta)d\Omega$$

$$\int S_1(\theta) 2\pi \sin\theta d\theta = 1$$ ⇐ Normalized single scattering function

$$S_m = S_{m-1}(\theta) \otimes S_1(\theta)$$

$$I(\theta)d\Omega = I_0 d\Omega \sum_{m=0}^{\infty} \frac{(t/\Lambda)^m}{m!} e^{-t/\Lambda} S_m(\theta)$$

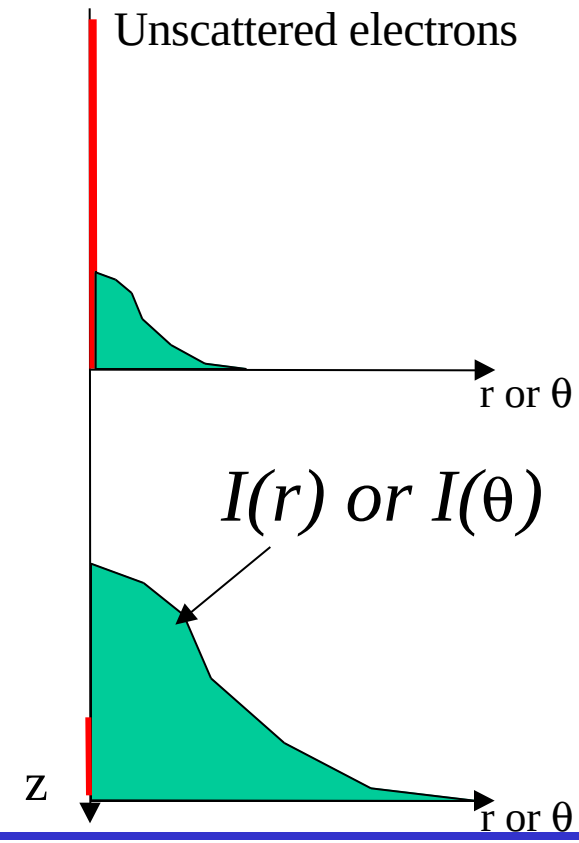
$$\overline{\theta^2} = \frac{t}{\Lambda}\overline{\theta_1^2}, \quad r = \theta z$$

Gaussian

Approximate $S_1(\theta) \approx e^{-\theta^2/\overline{\theta_1^2}}$, *assume* $I(r) \propto e^{-r^2/\overline{r^2}}$

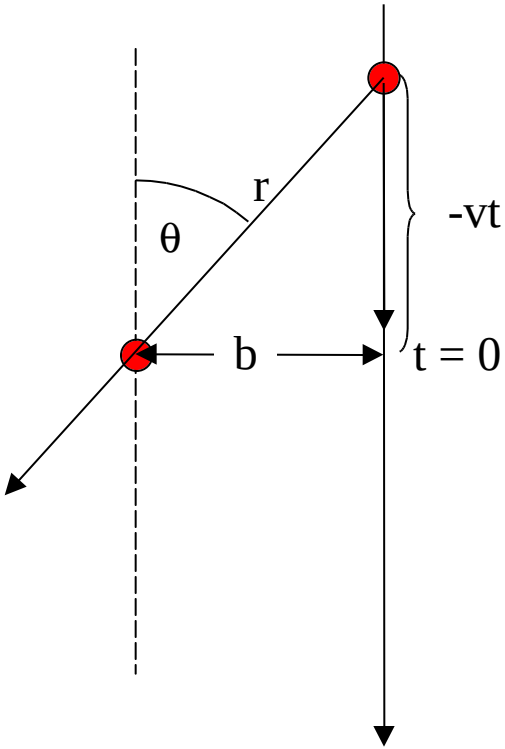
$$\overline{r^2} = \frac{2}{3}\frac{\overline{\theta_1^2}}{\Lambda} t^3, \quad r_{rms} \propto t^{3/2}$$

Appropriate for m< 25



Continuous Slowing Down Approximation

- Energy transfer, W ($<< E$, incident electron energy), occurs through Coulomb interaction between incident electrons and atomic electrons. Mean energy loss/path length is $-dE_m$. Stopping power, S:

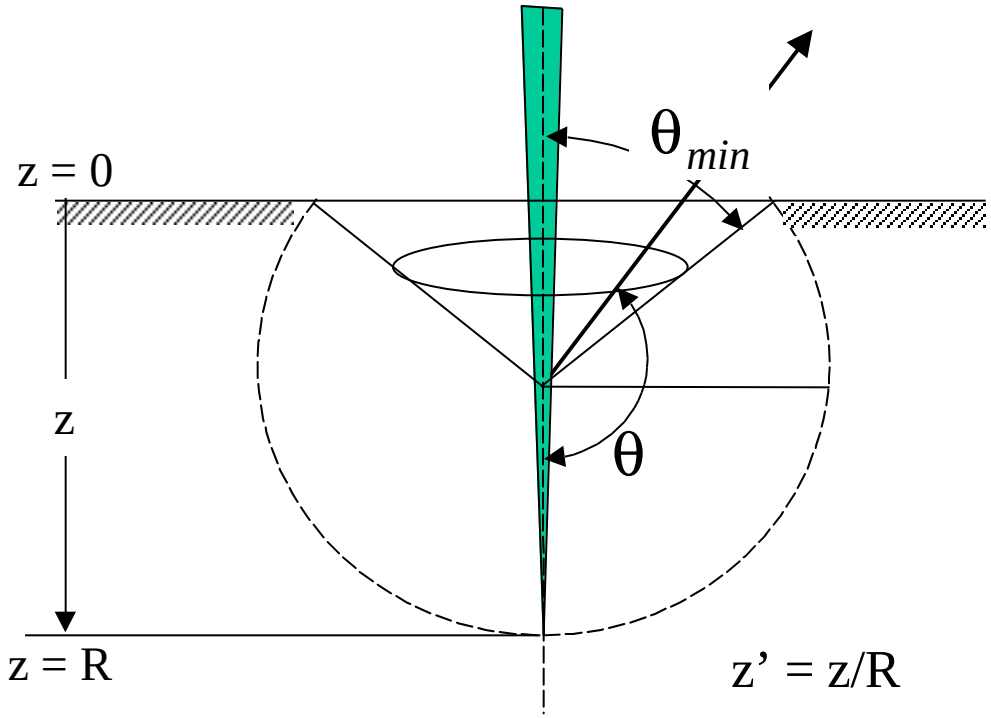


$$S = \left|\frac{dE_m}{ds}\right| = NZ\int_{W_{\min}}^{W_{\max}} \frac{d\sigma}{dW} W dW = \frac{2\pi e^4 N_A \rho Z}{(4\pi\varepsilon_0)^2 AE}\int_{b_{\min}}^{b_{\max}} \frac{db}{b}$$

$$= \frac{2\pi e^4 N_A \rho Z}{(4\pi\varepsilon_0)^2 AE}\ln\left(\frac{E}{J}\right), \quad J = 11.5Z, \quad Z \le 6$$

$$Range,\ R \propto E^n, \frac{1}{Z^m}, \quad 1.3 \le n \le 1.7, \quad m \approx 1$$

Diffusion - Everhart's Single Scattering Model

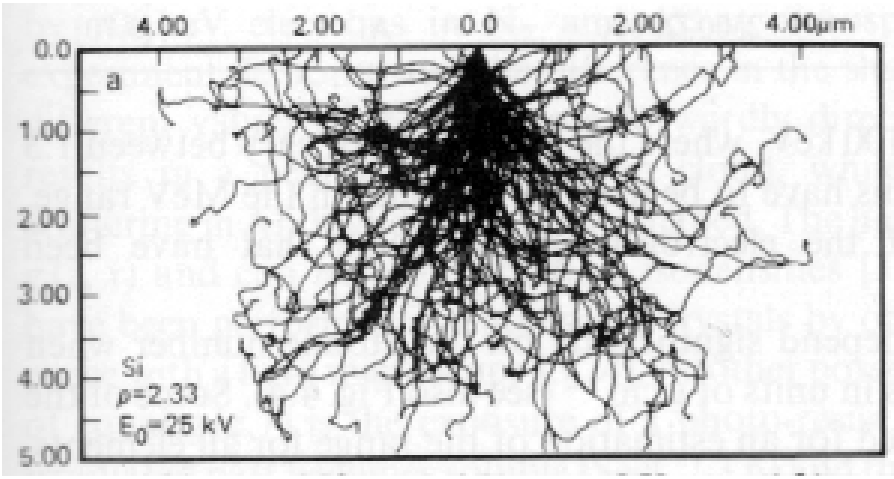


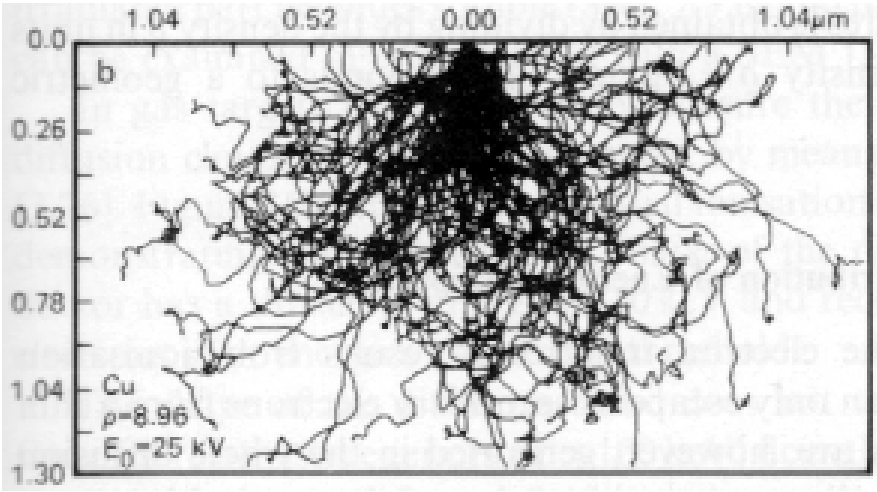
- Electron energy decreases with depth as $v = (c_T \rho z - v_0^4)^{1/4}$
- Intensity decreases as $dI(z) = N_A \rho \sigma \pi / (2A)\, I(z) dz$
- Electrons are backscattered by single scattering through angles $\pi - \theta_{min} < \theta < \pi$

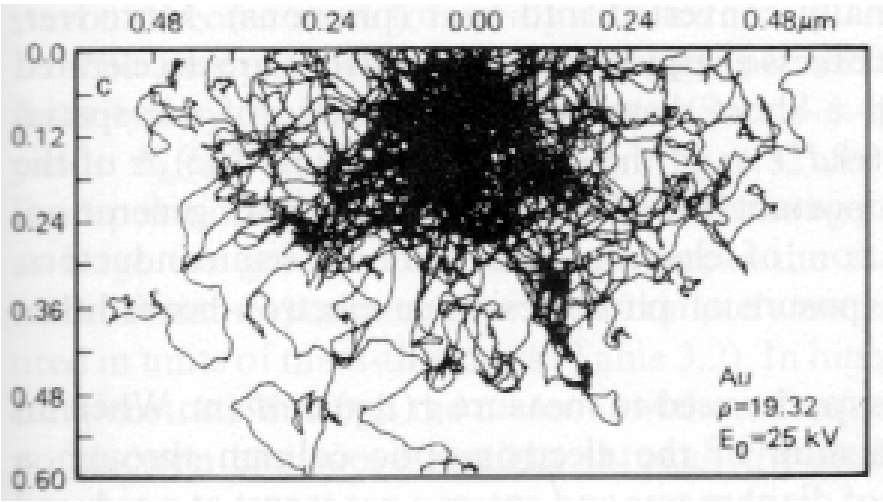
$$\eta = \int_0^{0.5} \frac{N_A \rho}{A I_0} \sigma(\theta_{\min}) I(z') dz' \approx \frac{0.012Z - 1 + 0.5^{0.012Z}}{0.012Z + 1}$$

η increases with increasing Z, R decreases with increasing Z. Character of proximity effect changes with atomic number

Monte-Carlo Simulation

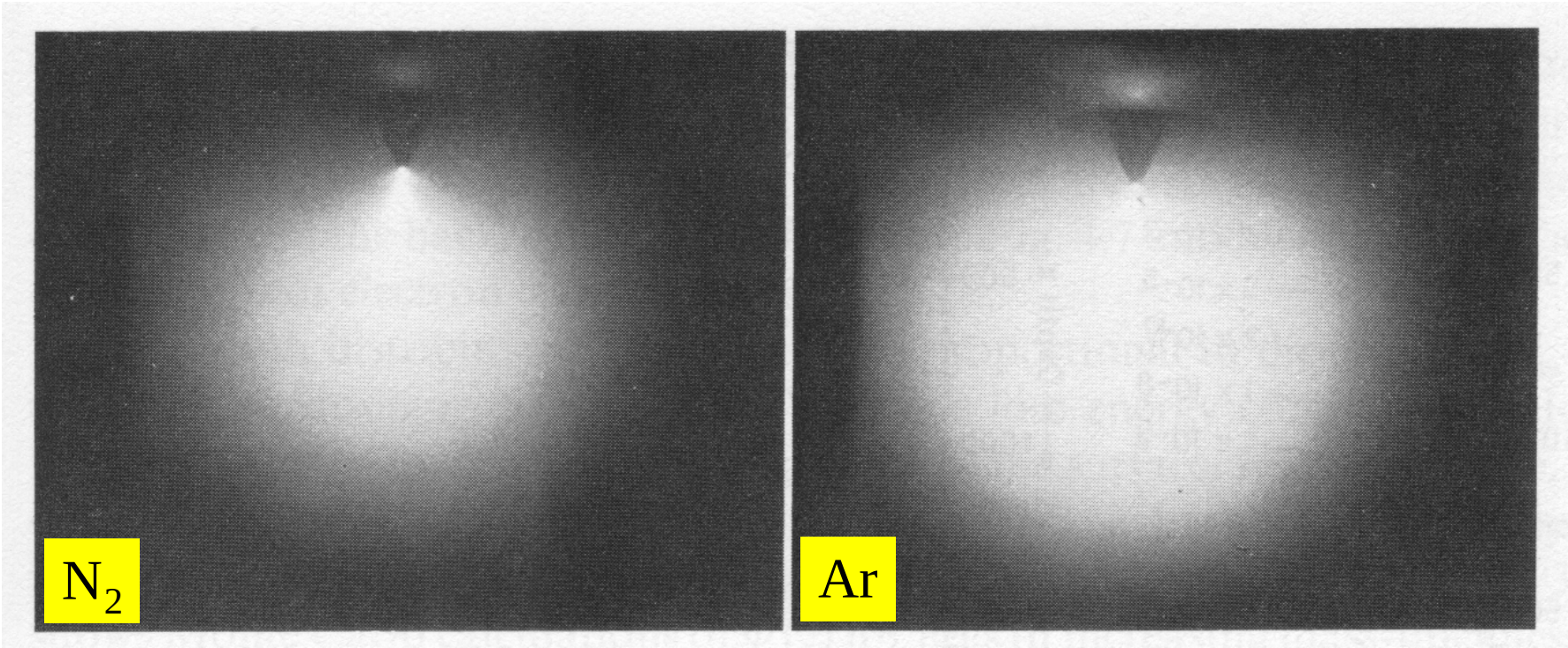






Note changes in horizontal and vertical scales as the atomic number increases: Si 14, Cu 29, Au 79

Scattering in Gas Targets



Ludwig Reimer, *"Scanning Electron Microscopy"*, Springer-Verlag (1985)

Inelastic Scattering

- Energy loss occurs through a variety of mechanisms
  - Molecular oscillations/phonons
    - ΔE = 20 meV - 1 eV
  - Conduction/valence electrons
    - Plasmons
    - Inter- or Intra-band transitions
    - ΔE = 1 eV - 50 eV
  - Core electrons
    - Ionization of inner shell electrons
      - X-rays
      - Auger electrons
    - ΔE_K = 110 eV (Be) - 80 keV (Au)

Described by dielectric theory - related to optical constants of material. Electron energy-loss spectra and those for light and x-rays are related

Energy loss in C is ≈ 0.24 eV/nm at 100 keV

Dielectric Theory

Optical constant : $\varepsilon = \varepsilon_1 + i\varepsilon_2 = (n + ik)^2$

Energy dissipation : $\dfrac{dW}{dt} = \mathbf{E.D}$

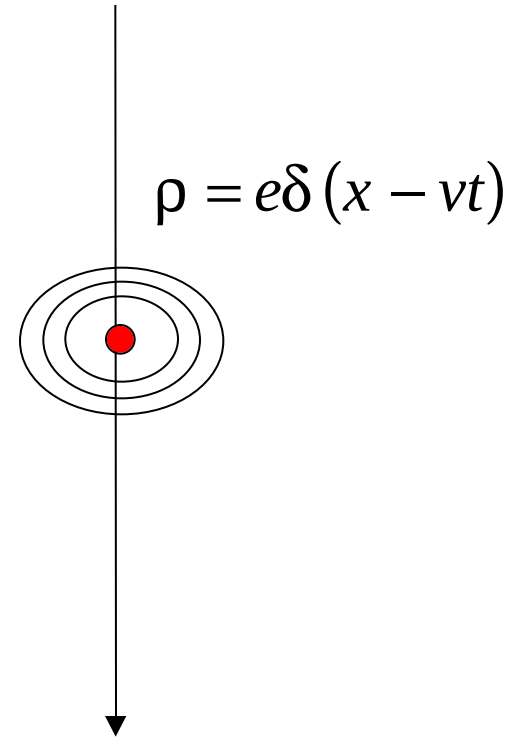
Electron : $\rho = e\delta(x - vt)$

$div\mathbf{D} = \rho, \quad \mathbf{E} = \mathbf{E}_0 e^{-i\omega t}, \quad \mathbf{D} = \varepsilon_0 \mathbf{E}$

$$\Rightarrow \frac{\overline{dW}}{dt} = \varepsilon_0 \varepsilon_2 \frac{\omega |\mathbf{E}_0|^2}{2} = \frac{1}{\varepsilon_0} \frac{\varepsilon_2}{|\varepsilon|^2} \frac{\omega D_0^{\,2}}{2} = \frac{1}{\varepsilon_0} \frac{\omega D_0^{\,2}}{2} \mathrm{Im}(-1/\varepsilon)$$

Note : $\varepsilon \rightarrow f(\omega)$

$$\frac{d^2\sigma}{d\Delta E d\Omega} = \frac{1}{\pi^2 a_H m v^2 N_e} \frac{\mathrm{Im}(-1/\varepsilon)}{(\theta^2 + \theta_E^2)}, \quad N_e = electrons / unit\ volume, \quad \theta_E = \frac{\Delta E}{2E}$$



$\sigma_{inel} \approx 20\ \sigma_{el}/Z$

20 eV loss at 100 keV gives a θ_E of 0.1 mrad

Plasmon & Optical Losses

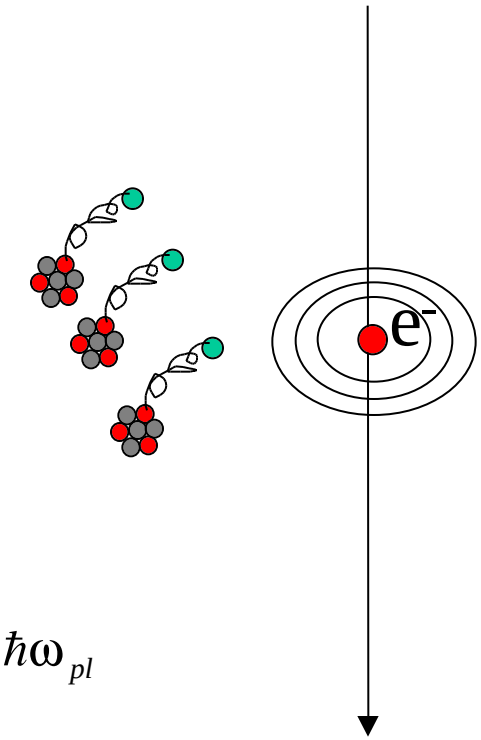
$$m^* \frac{d^2\mathbf{x}}{dt^2} + m^*\gamma \frac{d\mathbf{x}}{dt} = -e\mathbf{E}$$

$$\mathbf{x} = \frac{e}{m^*\omega^2} \frac{\omega^2 - i\omega\gamma}{\omega^2 + \gamma^2} \mathbf{E}$$

$$\mathbf{P} = -eN_e\mathbf{x} = \varepsilon_0 \chi_e \mathbf{E}, \quad \varepsilon = \varepsilon_0 (1 + \chi_e)$$

$$\Rightarrow \varepsilon_1 = \varepsilon_0 \left(1 - \frac{\omega_{pl}^2}{\omega^2} \frac{1}{1 + (\gamma/\omega)^2} \right), \quad \varepsilon_2 = \varepsilon_0 \frac{\gamma}{\omega} \frac{\omega_{pl}^2}{\omega^2} \frac{1}{1 + (\gamma/\omega)^2}$$

$$Plasma\ \ frequency: \omega_{pl} = \sqrt{\frac{N_e e^2}{\varepsilon_0 m^*}}, \quad Plasmon\ energy: \quad \Delta E_{pl} = \hbar\omega_{pl}$$



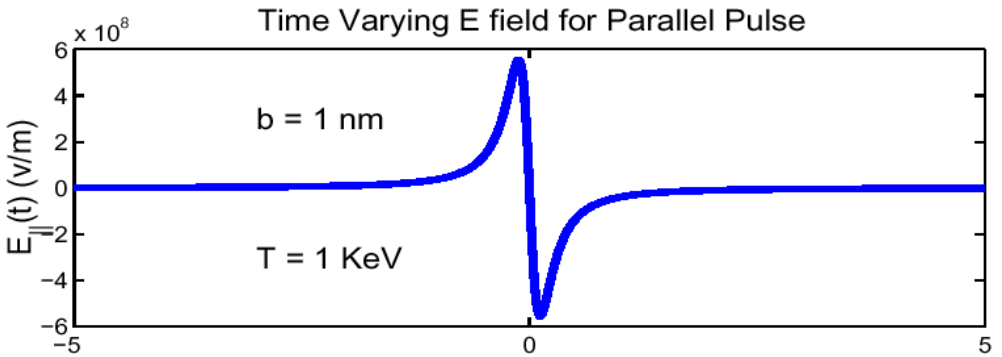
$$m^* \left(\frac{d^2\mathbf{x}}{dt^2} + \gamma \frac{d\mathbf{x}}{dt} + \omega_b^2 \mathbf{x} \right) = -e\mathbf{E}$$

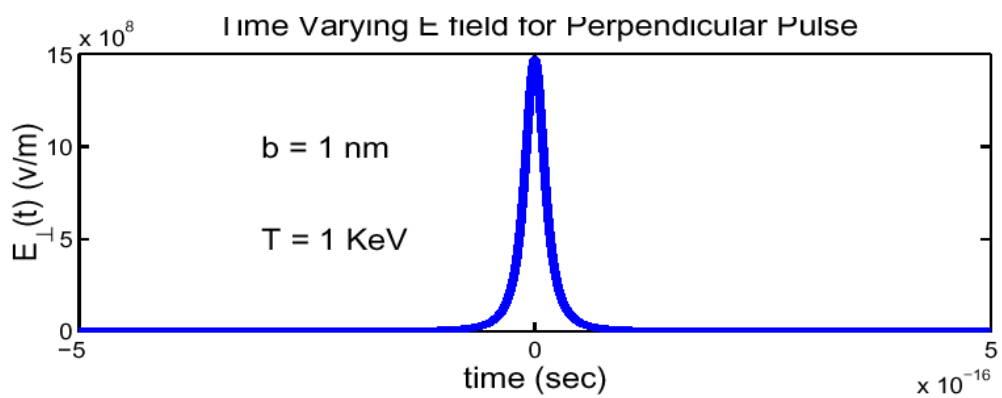
$$\varepsilon(\omega) = \varepsilon_0 \left(1 + \frac{N_e e^2}{\varepsilon_0 m^*} \frac{1}{\omega_b^2 - \omega^2 - i\omega\gamma} \right)$$

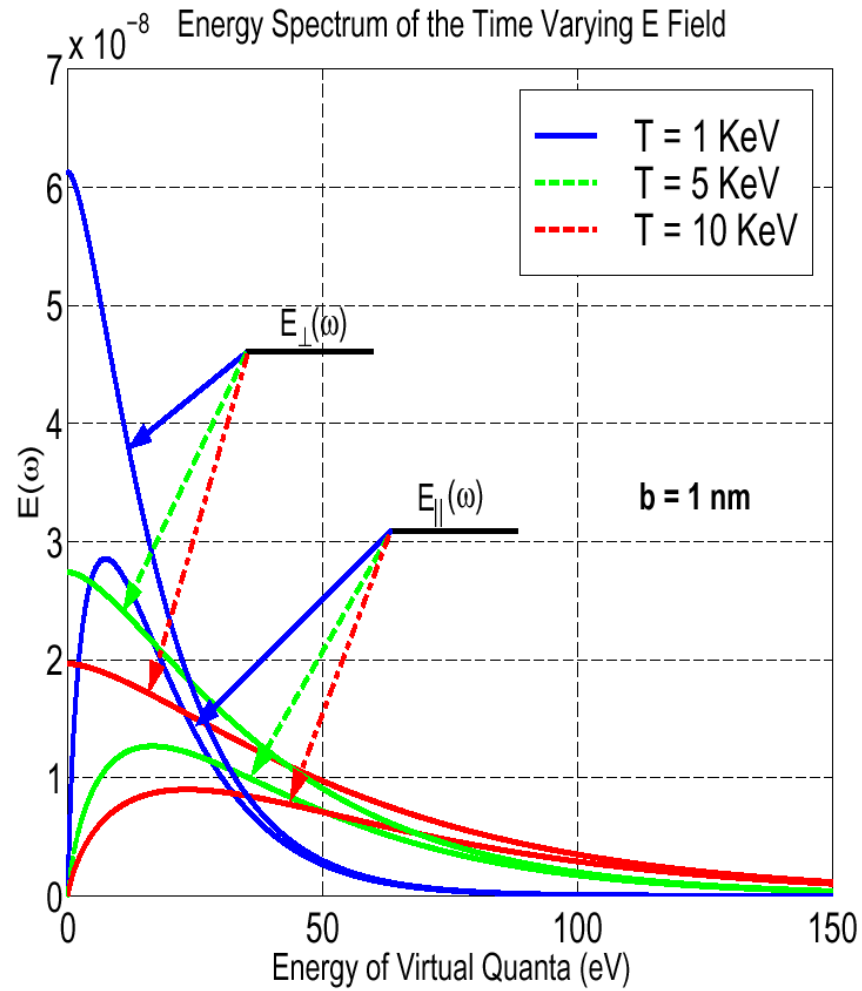
Introduce oscillators with other characteristic frequencies, ω_b , to represent bound electrons. Resonances occur when $\omega = \omega_b$. Passage of high-energy electron results in frequency pulse that can excite many resonances.

Lorentz oscillators: http://webphysics.davidson.edu/Projects/AnAntonelli/myThesis.html

Method of Virtual Quanta

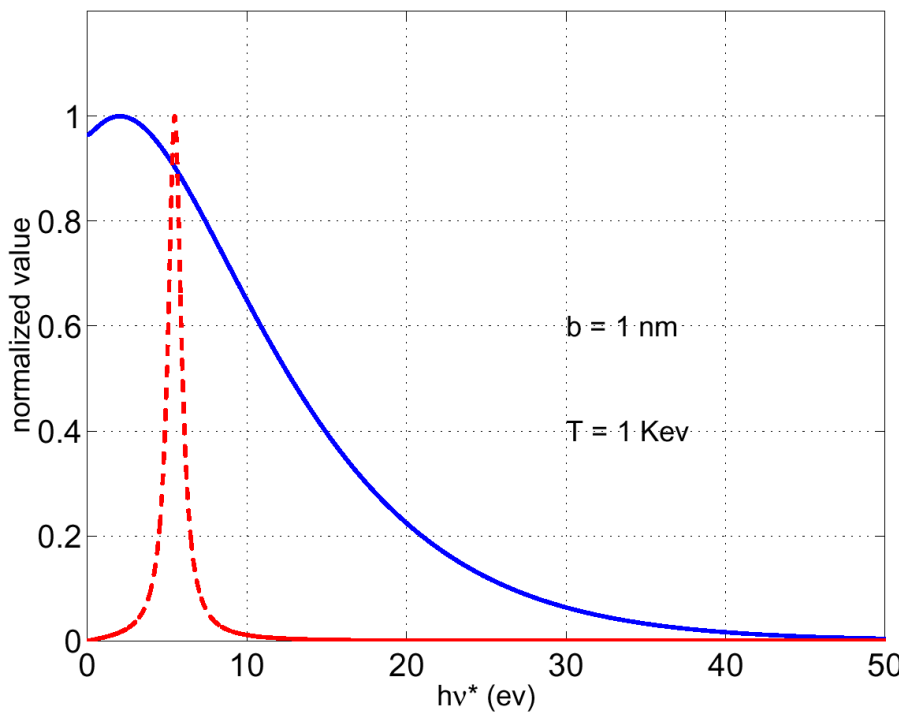


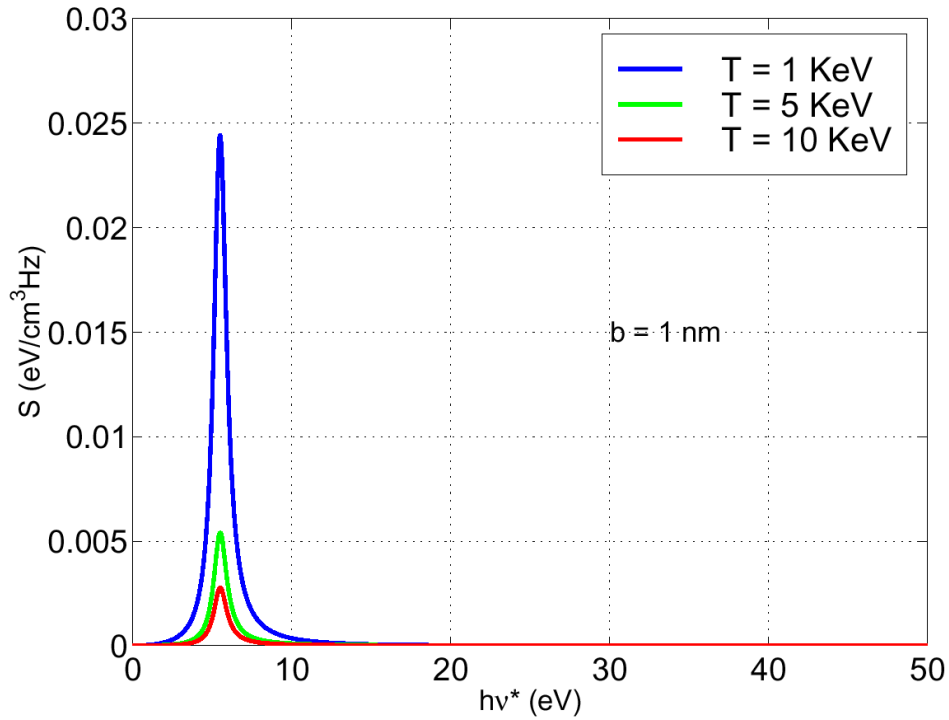




"Energy Transfer Between Electrons and Photoresist: Its Relation to Resolution", Geng Han and Franco Cerrina, J. Vac. Sci. Technol. B18 p3297 (2000)

Energy Transfer





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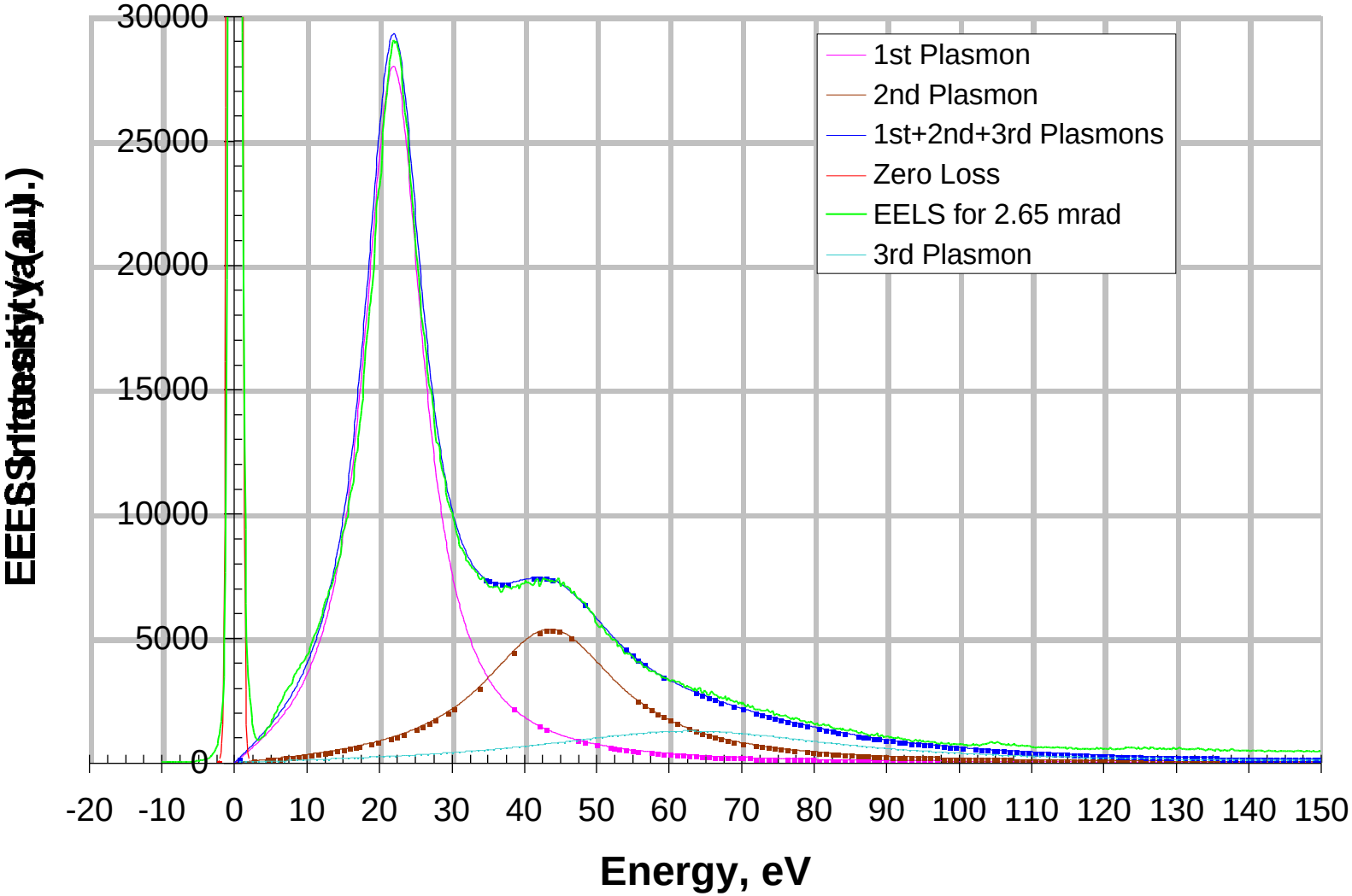
EELS Spectrum (SiN)